Validation exercises

Brightness of Light Experiment

In one experiment Saaty placed chairs at various distances (9, 15, 21, and 28 yards) from a light source in his back yard to see if his wife and young children, standing at the light source, could judge the relative brightness of the chairs.
The results achieved with pairwise verbal judgments (see Table 2) were in very close agreement with the inverse square law of optics, which says that relative brightness is inversely proportional to the square of the distance of the object from the light source.

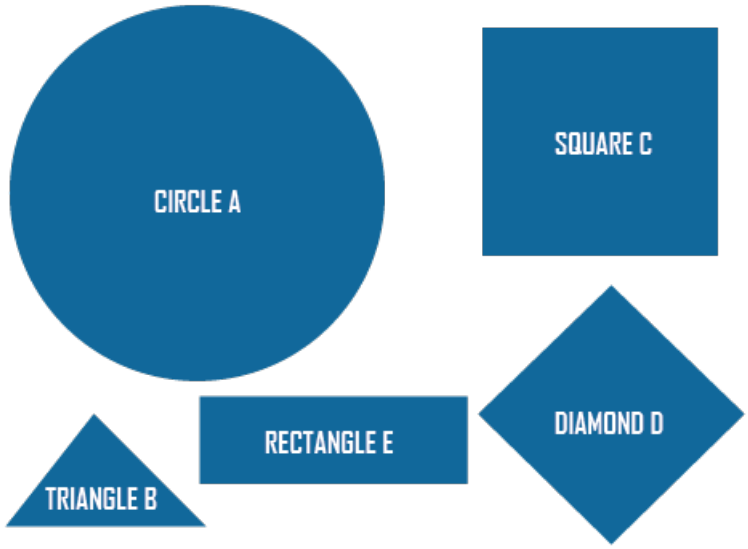
Table 2 – Results of Brightness of light on Chairs Experiment

Chair	Estimates from Wife's Judgments	Estimates from Sons' Judgments	Results from applying Inverse Square Law of Physics
1	0.61	0.62	0.61
2	0.24	0.22	0.22
3	0.10	0.10	0.11
4	0.05	0.06	0.06

Saaty *The Analytic Hierarchy Process*, p. 39.

Area Validation Experiment

The relative areas of different geometric shapes (each within an order of magnitude) were derived from pairwise verbal judgments for shapes such as the following:



This experiment has been performed thousands of times and you can easily do this individually or with a group. The actual relative areas and estimates derived from pairwise verbal judgments with a group of executives at the Ford Motor company is shown below:

Shape	Rank	Reverse	Proportion	Pairwise Verbal	Actual
Circle	1	5	33.3%	49.6%	47.5%
Triangle	5	1	6.7%	4.8%	4.9%
Square	2	4	26.7%	23.6%	23.2%
Rectangle	4	2	13.3%	14.5%	15.1%
Diamond	3	3	20.0%	7.5%	9.3%

Note how close the estimates are in the pairwise verbal column (column 5) to the actual column (column 6). Note also how deficient the estimates are if one were to simply derive the estimates based on the ranking (ordinal measures) of the shapes (column 4 above).

The accuracy of the derived ratio scale priorities are truly amazing considering that the inputs were ordinal measures (words on the fundamental verbal scale). Deriving ratio scale measures from ordinal inputs is somewhat magical since ratio measures have all of the information of ordinal measures, plus interval and ratio meaning as well. In a sense, it gives new meaning to GIGO -- garbage in, genius out! Do not, however, take this for granted. There may be cases where intervals or ratios of the priorities resulting from verbal judgments do not adequately represent the decision maker's feelings. It is incumbent upon the decision maker(s) to examine the resulting priorities and, if they do not adequately represent the decision maker's feelings, to revise the judgments in either the graphical or numerical mode.

If graphical or numerical judgments had been made, we would expect the results to be even better, since this is an "objective" problem with known scores. So why use verbal judgments at all?

Verbal judgments are often more appropriate when judging qualitative factors, and all crucial decisions have qualitative factors that must be evaluated. Numerical comparisons can convey the wrong sense of accuracy of input. If one were to say that clean air is 4.3 times more important than clean water, rather than between moderate and strongly more important, how does one defend the 4.3 rather than 4.2 or 4.6 or 5?