# Axioms of the Analytic Hierarchy Process

Originally, AHP was based on three relatively simple axioms.

The first axiom, the **reciprocal axiom**, requires that, if PC(EA,EB) is a paired comparison of elements A and B with respect to their parent, element C, representing how many times more the element A possesses a property than does element B, then PC(EB,EA) = 1/ PC(EA,EB). For example, if A is 5 times larger than B, then B is one fifth as large as A.

**Homogeneity**, the second axiom, states that complexity should be structured in a hierarchy of homogenous clusters. The elements to be compared within each cluster should be relatively homogenous, otherwise there would tend to be larger errors in judgment. When constructing a hierarchy of objectives, one should attempt to arrange elements in clusters so that with each cluster, they do not differ by more than an order of magnitude. (The AHP verbal scale ranges from 1 to 9, or about an order of magnitude. The numerical and graphical modes of AHP software implementations permit more than one order of magnitude, allowing a relaxation of this axiom, with the understanding that judgments beyond an order of magnitude generally result in a decrease in accuracy and increase in inconsistency.)

The third axiom, **hierarchic composition**, states that judgments about, or the priorities of, the elements in a hierarchy do not depend on lower level elements. This axiom is required for the principle of hierarchic composition to apply.

While the first two axioms are always consonant with real world applications, the hierarchic composition axiom requires careful examination, as it is not uncommon for it to be violated. While the preference for alternatives is almost always dependent on higher level elements - the objectives, the importance of the objectives might or might not be dependent on lower level elements - the alternatives. For example, in choosing a laptop computer, the relative importance of speed vs. weight might depend on the specific alternatives being considered—if the alternatives were about the same weight but differed greatly in speed, then speed might be considered to be more important. If this is the case, we say there is "feedback" from the alternatives to the objectives.

There are two basic ways to proceed in those situations where this axiom does not apply -- that is, when there is feedback. The first involves a formal application of feedback and a super matrix calculation for synthesis rather than hierarchic composition. This approach is called the Analytic Network Process. For simple feedback (between adjacent levels only), this is equivalent to deriving priorities for the objectives with respect to each alternative, in addition to deriving priorities for the alternatives with respect to each objective. The resulting priorities are processed in a super matrix, which is equivalent to the convergence of iterative hierarchical compositions.

While this approach is extremely powerful and flexible (feedback within levels and between nonadjacent levels can also be accommodated), a simpler approach that usually works well is to make judgments for lower levels of the hierarchy first (or to reconsider judgments at the upper levels after making judgments at the lower level). In so doing, the brain performs the feedback function by considering what was learned at lower levels of the hierarchy when making judgments for upper levels. Thus, an important rule of thumb is to make judgments in a hierarchy from the bottom up, unless one is sure that there is no feedback, or one already has a good understanding of the alternatives and their trade-offs. Even if this is not done, adherence to AHP's fourth axiom (below) as well as the process notion of AHP, can usually lead to appropriate judgments, since an examination of the priorities after a first iteration of the model will highlight those areas where judgments should be revised based on what has been learned.

A fourth axiom, introduced later by Saaty, says that individuals who have reasons for their beliefs should make sure that their ideas are adequately represented for the outcome to match these expectations. While this axiom might sound a bit vague, it is very important because the generality of AHP makes it possible to apply AHP in a variety of ways and adherence to this axiom prevents applying AHP in inappropriate ways. We will illustrate this a bit later.

The simplicity and generality of AHP fit nicely with Ockham's razor which contends that the simplest of two or more competing theories is preferable. Not only are AHP's axioms simpler and more realistic than other decision theories, but the ratio scale measures that it produces makes AHP more powerful as well.