Ideal and Distributive Synthesis: Closed and Open Systems

Originally, AHP had only one synthesis mode – later called the "distributive" synthesis mode. A distributive synthesis distributes priorities from the goal down through the alternatives and is analogous to dividing priorities in a pie chart, which is intuitive for decision makers to comprehend. The sum of the global priorities for each alternative with respect to each covering objective represents the overall priority of that alternative. The priorities have ratio scale properties (as well as, of course, interval and ordinal properties), which means that they can be used in making a choice or in allocating resources. This synthesis operation can be thought of as distributing the goals priority of 1.0 to the alternatives under consideration, and is today called the distributive synthesis mode. Originally, this was the only synthesis mode of AHP. Critics of AHP pointed out situations where a different synthesis mode is more appropriate.

Ideal Synthesis

The ideal synthesis mode of AHP was introduced by Forman in 1993 [Forman, E. H., 1993. "Ideal and Distributed Synthesis Modes for the Analytic Hierarchy Process," *George* Washington University Working Paper] and was incorporated into Expert Choice software, for situations where the introduction of new alternatives to a decision might - using the distributive synthesis mode - cause the ranks of existing alternatives to change, even when none of the judgments with respect to the original alternatives are changed. There are some situations where it is reasonable for the ranks of existing alternatives to change, and others where it is not.

Forman and Gass defined these as *closed* and *open* systems which exhibit the properties of scarcity and abundance. The distribution of a country's gold is an example of a *closed system*, where the distributive mode (described above) makes sense. Consider the situation where a newly formed country has decided to distribute its gold reserve to identified segments of the society based on criteria that includes population and economic potential. After distributing the gold, a previously overlooked segment of the society with a small population but great potential is added to the allocation. To make a distribution to this new segment, gold would have to be taken back from the old segments and redistributed. Depending on the weights for the criteria, sectors that were highly populated but with relatively low economic potential could lose less than those with relatively low population and a high economic potential causing a reversal in ranks of these pre-existing segments to take place – a reasonable consequence. Similarly, an AHP model might be used to forecast that the Democratic candidate in a U.S. Presidential race is slightly ahead of the Republican candidate. Now suppose an Independent candidate enters the race. It is conceivable (and in fact has happened in more than one recent Presidential race) that the ranks of the Democratic and Republican candidates would change since the Independent candidate might take more votes from the Democratic candidate than the Republication candidate.

Now suppose that you were considering the purchase of a new computer and have already identified 5 alternatives to choose from. Further assume that alternative "A" was, according to a multi-objective decision model such as AHP, the most preferred, with alternative "B" a very close second, in large part, due to the fact that alternative "A" was preferred on cost. Now suppose that someone suggest that you consider a 6^{th} alternative, alternative "F" that is relatively inexpensive, but not as inexpensive as alternative "A." Furthermore, alternative "F" is not as preferable as "A" or any other alternative on any of the objectives. If one were to use the AHP distributive mode, it is possible (although not common) for the ranks of alternatives "A" and "B" to change when alternative "F" is introduced – simply because the alternative "F" would take more priority from "A" on cost than it would from alternative "B" – and this is enough to cause the ranks of alternatives "A" and "B" to reverse. Proponents of MAUT had criticized AHP because of this phenomenon. On one hand, this criticism was justified because in situations like this, one would not want to have their chosen alternative susceptible to a change when an "irrelevant" alternative was introduced into consideration. On the other hand, there is a need for a prioritization methodology like AHP to allow for such rank reversals in situations where they do make sense -- as described in the closed system above.

Rank reversal

To address the need for a prioritization method to be applicable to both situations, a second synthesis mode, called the Ideal synthesis, was added to AHP and Expert Choice. Rather than distributing the priority of each covering objective to the alternatives based on the local priorities of the alternatives, the AHP ideal synthesis mode assigns all of each covering objectives priority to the alternative that is most preferred under that objective, and assigns priorities to the other alternatives under each covering objective in proportion to their priority, relative to the most preferred alternative under that covering objective. The sum of the priorities is then normalized, so they add to one. This approach is called the "open system" approach since the introduction of new alternatives will require more priority to be assigned under each covering objective but will not change the proportionality (or ranks) of existing alternatives. The result then, is that there can be no rank reversals when new alternatives are added (or existing alternatives removed) from an evaluation. We will say more about this in a later section of the article.

To summarize, AHP has two synthesis modes: distributive synthesis, appropriate to "closed systems'"where there is a fixed resource to be considered – such as votes in an election or market for a product, or estimating forecast probabilities; and ideal synthesis, appropriate to "open systems" where introduction of new alternatives can be modeled by adding an additional "priority," and then normalizing – thus maintaining the ratios and ranks of existing alternatives when new alternatives are added for consideration (or existing alternatives removed from consideration).