Levels Of Measurement

Not all numbers have the same meaning in terms of what operations can be legitimately performed on them. In order for operations like addition and multiplication with numbers to be mathematically meaningful, the numbers must possess certain properties.

Nominal, ordinal, interval, and ratio

A widely used categorization for levels of measurement is due to S. S Stevens, in which four levels of measurement are defined:

- nominal,
- ordinal,
- interval, and
- ratio.

The meaning of information contained in each level increases, or we might say that each level contains the information in all lower level plus additional information.

Nominal means in name only; e.g., the number of a horse in a horse race designates the horse, not the position at the finish. (Although, as Stevens points out, the level of measurement is sometimes not clear from the number itself. The number on the horse does designate the starting gate position – but this information is not important for most people.)

Ordinal numbers add information about order or ranks – either high-to-low, or low-to-high. For example, horse number 3 finished in 1st place.

Interval numbers add information about the "intervals" between measures. For example, Horse number 3 finished in first place, 3 lengths or 12 meters ahead of the second place horse.

Ratio numbers have the highest level of measurement where the ratios of the measures are meaningful in terms of ratios or proportions. For example, the finish time for the winning horse was .973 times that of the second-place horse.

© 2019 Expert Choice. All Rights Reserved

The "appropriate" measure for a specific application depends on the application; and a decision maker has, in some circumstances, the discretion to use whatever is most meaningful to him/her. For example, the intervals in time between the first- and second-place horses in a previous race might be more informative of a horse than the ratios of the finishing times. However, there are circumstances where the choice of level of measurement is not discretionary – that is, if results that are deemed to be mathematically meaningful are desired.

Mathematical definitions vs. practical understanding

Mathematicians define an interval scale as being invariant under the transformation y = ax + b, and a ratio transformation as being invariant under the transformation y = ax. While this is precise, it doesn't convey an understanding of the practical meanings of interval and ratios scales, which are:

Corresponding intervals anywhere on an interval scale have the same meaning. For example, the interval between 1 and 2 has the same meaning (significance) as the interval between 98 and 99. That is, intervals on an interval scale are meaningful.

Corresponding ratios anywhere on a ratio scale have the same meaning. For example, the ratio of 6 to 3 has the same meaning as the ratio of 100 to 50. That is, ratios on a ratio scale are meaningful.

Distance, for example, is a ratio scale. The ratio of two miles to one mile has the same meaning as the ratio of 100 miles to 50 miles – the first is twice the distance of the second. If we multiply miles by some constant (e.g., to convert to meters or feet), the ratios will still be two to one. However, if we add a constant (say 5), then the ratios will no longer be the same, as the ratio of 7 miles to 5 miles is not the same as the ratio of 105 miles to 55 miles.

There is a misconception that a ratio scale has to possess a true zero. This is not the case. A ratio scale has an implied zero, but not necessarily a true zero. For example, while the measures of areas of geometric shapes are a ratio scale measure, an actual shape with zero area does not exist.

Why are levels of measure important

Quite simply, in order for mathematical operations with numbers to be "meaningful," the numbers must possess certain levels of measurement. For example, care must be taken not to multiply or add ordinal measures. It is meaningful to multiply ratio scale numbers, or a ratio scale number by an interval scale number, but it is mathematically incorrect to multiply two interval level numbers.

Why ratio scale measures are important

There are several reasons why ratio scale measures, as derived by using AHP, are important. When structuring a complex decision (or forecasting model) as a hierarchy of objectives, with sub-objectives, and sub-sub objectives, we derive priorities for the elements in the top cluster, clusters below this cluster, and so on. If we were to multiply the priorities of the elements in each cluster by the priorities of the cluster above (we call the priorities of the elements in each cluster "local" priorities) to derive what we refer to as "global" priorities of the elements themselves, then the results would be mathematically meaningless unless the priorities possessed the ratio scale property.

As far as the level of measure of the alternative priorities, these too would be ratio scale measures provided the alternative priorities with respect to the lowest level objectives in the hierarchy are ratio scale priorities. However, using

© 2019 Expert Choice. All Rights Reserved

only interval measures as with other methods such as Multi Attribute Utility Theory (MAUT), we could still multiply by the ratio scale priorities of the lowest level sub-objectives and derive mathematically meaningful results. The results would, however, possess only the interval and not ratio level measure property. This is adequate for ascertaining which alternative has the highest overall priority, but is not adequate if we want to use the derived priorities in places where ratio scale priorities (i.e., proportion) are required for making optimal decisions, such as allocating resources to a portfolio of projects/alternatives.

There are some applications where the priorities of one evaluation are multiplied by priorities of another evaluation. For example, the relative risks of possible occurrences can be determined by deriving the relative impacts of the occurrences, and multiplying by the relative likelihoods of the occurrences. Risks derived in this manner are mathematically meaningful only if the measures of relative impact and relative likelihood are both ratio scale measures.

Ratio scale priorities are required in any application where meaningful measures of proportionality are assumed - for example, when allocating resources to a set of alternatives where the anticipated benefits are assumed to be proportional to the measured priorities of the alternatives.

© 2019 Expert Choice. All Rights Reserved